Synthesis of Band-Limited Orthogonal Signals for Multichannel Data Transmission

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This paper presents a principle of orthogonal multiplexing for transmitting a number of data messages simultaneously through a linear band-limited transmission medium at a maximum data rate without interchannel and intersymbol interferences. A general method is given for synthesizing an infinite number of classes of band-limited orthogonal time functions in a limited frequency band. Stated in practical terms, the method permits the synthesis of a large class of practical transmitting filter characteristics for an arbitrarily given amplitude characteristic of the transmission medium. Rectangular-shaped ideal filters are not required. The synthesis procedure is convenient. Furthermore, the amplitude and the phase characteristics of the transmitting filters can be synthesized independently. Adaptive correlation reception can be used for data processing, since the received signals remain orthogonal no matter what the phase distortion is in the transmission medium. The system provides the same signal distance protection against channel noises as if the signals of each channel were transmitted through an independent medium and intersymbol interference in each channel were eliminated by reducing data rate.

I. INTRODUCTION

In data transmission, it is common practice to operate a number of AM data channels through a single band-limited transmission medium. The system designer is faced with the problem of maximizing the overall data rate, and minimizing interchannel and intersymbol interferences. In certain applications, the channels may operate on equally spaced center frequencies and transmit at the same data rate, and the signaling intervals of different channels can be synchronized. For these applications, orthogonal multiplexing techniques can be considered. Several orthogonal-multiplexed systems developed[1,2] in the past use special sets of time-limited orthogonal signals. These signals have widely spread frequency spectra (e.g., a sin x/x spectrum). Consequently, when these signals are transmitted through a band-limited transmission medium at a data rate equivalent to that proposed in this paper, certain portions of the signal spectrum will be cut off and interferences will take place. For instance, the interferences because of band-limitation have been computed[3] for a system using time-limited orthogonal sine and cosine functions.

This paper shows that by using a new class of band-limited orthogonal signals, the AM channels can transmit through a linear band-limited transmission medium at a maximum possible data rate without inter-channel and intersymbol interferences. A general method is given for synthesizing an infinite number of classes of band-limited orthogonal time functions in a limited frequency band. This method permits one to synthesize a large class of transmitting filter characteristics for arbitrarily given amplitude and phase characteristics of the transmission medium. The synthesis procedure is convenient. Furthermore, the amplitude and the phase characteristics of the transmitting filters can be synthesized independently, i.e., the amplitude characteristics need not be altered when the phase characteristics are changed, and vice versa. The system can be used to transmit not only binary digits (as in Ref. 1) or m-ary digits (as in Ref. 2), but also real numbers, such as time samples of analog information sources. As will be shown, the system satisfies the following requirements.

(*i*) The transmitting filters have gradual cutoff amplitude characteristics. Perpendicular cutoffs and linear phases are not required.

(*ii*) The data rate per channel is $2f_s$ bauds,* where f_s is the center frequency difference between two adjacent channels. Overall data rate of the system is $[N/(N+1)]\, R_{\max}$, where N is the total number of channels and $R_{\max}$, which equals two times the overall baseband bandwidth, is the Nyquist rate for which unrealizable rectangular filters with perpendicular cutoffs and linear phases are required. Thus, as N increases, the overall data rate of the system approaches the theoretical maximum rate $R_{\max}$, yet rectangular filtering is not required.

(*iii*) When transmitting filters are designed for an arbitrary given amplitude characteristic of the transmission medium, the received signals remain orthogonal for all phase characteristics of the transmission medium. Thus, the system (orthogonal transmission plus adaptive

* The speed in bauds is equal to the number of signal digits transmitted in one second.

correlation reception) eliminates interchannel and intersymbol interferences for all phase characteristics of the transmission medium.

(*iv*) The distance in signal space between any two sets of received signals is the same as if the signals of each AM channel were transmitted through an independent medium and intersymbol interference in each channel were eliminated by reducing data rate. The same distance protection is therefore provided against channel noises (impulse and Gaussian noise). For instance, for band-limited white Gaussian noise, the receiver receives each of the overlapping signals with the same probability of error as if only that signal were transmitted. The distances in signal space are also independent of the phase characteristics of the transmitting filters and the transmission medium.

(*v*) When signaling intervals of different channels are not synchronized, at least half of the channels can transmit simultaneously without interchannel or intersymbol interference.

II. ORTHOGONAL MULTIPLEXING USING BAND-LIMITED SIGNALS

Consider N AM data channels sharing a single linear transmission medium which has an impulse response $h(t)$ and a transfer function $H(f) \exp [J\eta(f)]$ (see Fig. 1).* $H(f)$ and $\eta(f)$ will be referred to, respectively, as the amplitude and the phase characteristic of the transmission medium.

Since this analysis treats only transmission media having linear properties, the question of performance on real channels subject to such impairments as nonlinear distortion and carrier frequency offset is not considered here. Such considerations are subjects of studies beyond the scope of the present paper.

In deriving the following results, it is not necessary to assume that the transmitting filters and data processors operate in baseband. However, this assumption will be made since in practice signal shaping and data processing are usually performed in baseband. Carrier modulation and demodulation (included in the transmission medium) can be performed by standard techniques and need not be specified here.

Consider a single channel first (say, the ith channel). Let b_0, b_1, b_2, $\cdots$, be a sequence of m-ary ($m \geqq 2$) signal digits or a sequence of real numbers to be transmitted over the ith channel. As is well known,[4] b_0, b_1, b_2, $\cdots$ can be assumed to be represented by impulses with proportional heights. These impulses are applied to the ith transmitting filter at the rate of one impulse every T seconds (data rate per channel

* J denotes the imaginary number $\sqrt{-1}$, while j is used as an index.

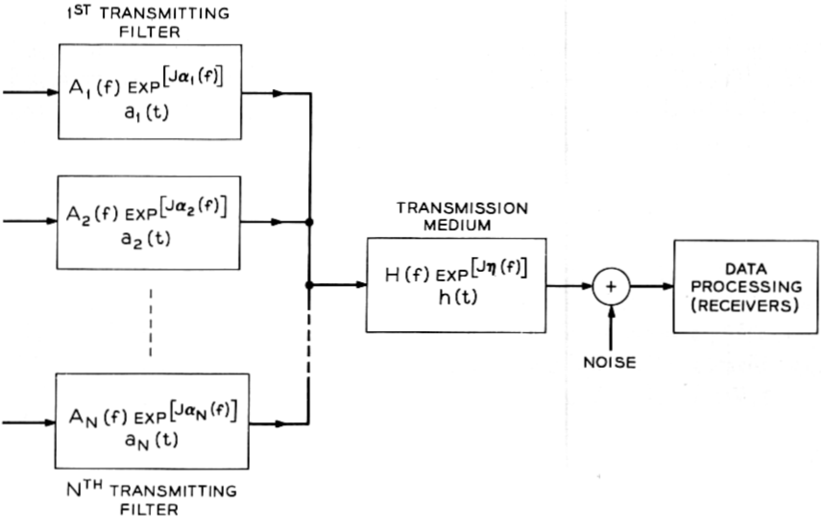

Fig. 1 — N data channels transmitting over one transmission medium.

equals $1/T$ bauds). Let $a_i(t)$ be the impulse response of the ith transmitting filter, then the ith transmitting filter transmits a sequence of signals as

$$b_0 \cdot a_i(t) + b_1 \cdot a_i(t - T) + b_2 \cdot a_i(t - 2T) + \cdots .$$

The received signals at the output of the transmission medium are

$$b_0 \cdot u_i(t) + b_1 \cdot u_i(t - T) + b_2 \cdot u_i(t - 2T) + \cdots ,$$

where

$$u_i(t) = \int_{-\infty}^{\infty} h(t - \tau) a_i(\tau) \, d\tau.$$

These received signals overlap in time, but they are orthogonal if

$$\int_{-\infty}^{\infty} u_i(t) u_i(t - kT) \, dt = 0, \qquad k = \pm 1, \pm 2, \cdots . \tag{1}$$

As is well known, orthogonal signals can be separated at the receiver by correlation techniques;* hence, intersymbol interference in the ith channel can be eliminated if (1) is satisfied.

Next consider interchannel interference. Let c_0, c_1, c_2, $\cdots$ be the

* Correlation reception and its adaptive feature will be briefly discussed in Appendix C.

m-ary signal digits or real numbers transmitted over the jth channel which has impulse response $a_j(t)$. It has been assumed in Section I that the channels transmit at the same data rate and that the signaling intervals of different channels are synchronized, hence the jth transmitting filter transmits a sequence of signals,

$$c_0 \cdot a_j(t) + c_1 \cdot a_j(t - T) + c_2 \cdot a_j(t - 2T) + \cdots .$$

The received signals at the output of the transmission medium are

$$c_0 \cdot u_j(t) + c_1 \cdot u_j(t - T) + c_2 \cdot u_j(t - 2T) + \cdots .$$

These received signals overlap with the received signals of the ith channel, but they are mutually orthogonal (no interchannel interference) if

$$\int_{-\infty}^{\infty} u_i(t) u_j(t - kT)\, dt = 0, \qquad k = 0, \pm 1, \pm 2, \cdots . \tag{2}$$

Thus, intersymbol and interchannel interferences can be simultaneously eliminated if the transmitting filters can be designed (i.e., if the transmitted signals can be designed) such that (1) is satisfied for all i and (2) is satisfied for all i and j ($i \neq j$).

Denote $U_i(f) \exp [J\mu_i(f)]$ as the Fourier transform of $u_i(t)$. One can rewrite (1) as

$$\int_{-\infty}^{\infty} U_i^2(f) \exp(-J2\pi fkT)\, df = 0 \qquad k = \pm 1, \pm 2, \cdots \qquad i = 1, 2, \cdots, N, \tag{3}$$

and rewrite (2) as

$$\int_{-\infty}^{\infty} U_i(f) \exp [Ju_i(f)] U_j(f) \exp [-J\mu_j(f)] \cdot \exp [-J2\pi fkT]\, df = 0 \qquad k = 0, \pm 1, \pm 2, \cdots \qquad i,j = 1, 2, \cdots, N \qquad i \neq j. \tag{4}$$

Let $A_i(f) \exp [J\alpha_i(f)]$ be the Fourier transform of $a_i(t)$. The transfer function of the transmission medium is $H(f) \exp [J\eta(f)]$. Equation (3) becomes

$$\int_{-\infty}^{\infty} A_i^2(f) H^2(f) \exp(-J2\pi fkT)\, df = 0 \qquad k = \pm 1, \pm 2, \cdots \qquad i = 1, 2, \cdots, N, \tag{5}$$

or

$$\int_0^{\infty} A_i^2(f) H^2(f) \cos 2\pi fkT\, df = 0 \qquad k = 1, 2, 3, \cdots \qquad i = 1, 2, \cdots, N. \tag{6}$$

Equation (4) becomes

$$\int_{-\infty}^{\infty} A_i(f) A_j(f) H^2(f) \exp\{J[\alpha_i(f) - \alpha_j(f) - 2\pi fkT]\}\, df = 0 \qquad k = 0, \pm 1, \pm 2, \cdots \qquad i,j = 1, 2, \cdots, N \qquad i \neq j. \tag{7}$$

Writing (7) in real and imaginary parts and comparing parts for $k = 1, 2, \cdots$ and $k = -1, -2, \cdots$, it is seen that (7) holds if and only if

$$\int_0^{\infty} A_i(f) A_j(f) H^2(f) \cos[\alpha_i(f) - \alpha_j(f)] \cos 2\pi fkT\, df = 0, \tag{8}$$

and

$$\int_0^{\infty} A_i(f) A_j(f) H^2(f) \sin[\alpha_i(f) - \alpha_j(f)] \sin 2\pi fkT\, df = 0, \tag{9}$$

where

$$k = 0, 1, 2, \cdots \qquad i,j = 1, 2, \cdots, N \qquad i \neq j.$$

It will be recalled that the transmitting filters and the data processors operate in baseband. Let f_i, $i = 1, 2, \cdots, N$, denote the equally spaced baseband center frequencies of the N independent channels. One can choose

$$f_1 = (h + \tfrac{1}{2}) f_s, \tag{10}$$

where h is any positive integer (including zero), and f_s is the difference between center frequencies of two adjacent channels. Thus,

$$f_i = f_1 + (i - 1)f_s = (h + i - \tfrac{1}{2})f_s . \tag{11}$$

Carrier modulation will translate the baseband signals to a given frequency band for transmission.

Each AM data channel transmits at the data rate $2f_s$ bauds. Hence,

$$T = \frac{1}{2f_s} \text{ seconds.} \tag{12}$$

For a given amplitude characteristic $H(f)$ of the transmission medium, band-limited transmitting filters can be designed (i.e., band-limited transmitted signals can be designed) such that (6), (8), (9), and (12) are simultaneously satisfied (no intersymbol and interchannel interference for a data rate of $2f_s$ bauds per channel). In addition, the five requirements in Section I are also satisfied. A general method of designing these transmitting filters is given in the following theorem.

Theorem: For a given $H(f)$, *let* $A_i(f)$, $i = 1, 2, \cdots, N$, *be shaped such that*

$$\begin{aligned} A_i^2(f)H^2(f) &= C_i + Q_i(f) > 0, \qquad f_i - f_s < f < f_i + f_s \\ &= 0, \qquad f < f_i - f_s , \qquad f > f_i + f_s , \end{aligned} \tag{13}$$

where C_i *is an arbitrary constant and* $Q_i(f)$ *is a shaping function having odd symmetries about* $f_i + (f_s/2)$ *and* $f_i - (f_s/2)$, *i.e.,*

$$Q_i\left[\left(f_i + \frac{f_s}{2}\right) + f'\right] = -Q_i\left[\left(f_i + \frac{f_s}{2}\right) - f'\right], \qquad 0 < f' < \frac{f_s}{2}, \tag{14}$$

$$Q_i\left[\left(f_i - \frac{f_s}{2}\right) + f'\right] = -Q_i\left[\left(f_i - \frac{f_s}{2}\right) - f'\right], \qquad 0 < f' < \frac{f_s}{2}. \tag{15}$$

Furthermore, the function $[C_i + Q_i(f)]\cdot[C_{i+1} + Q_{i+1}(f)]$ *is an even function about* $f_i + (f_s/2)$, *i.e.,*

$$\begin{aligned} &\left[C_i + Q_i\left(f_i + \frac{f_s}{2} + f'\right)\right]\left[C_{i+1} + Q_{i+1}\left(f_i + \frac{f_s}{2} + f'\right)\right] \\ &\qquad = \left[C_i + Q_i\left(f_i + \frac{f_s}{2} - f'\right)\right]\left[C_{i+1} + Q_{i+1}\left(f_i + \frac{f_s}{2} - f'\right)\right] \\ &\qquad\qquad 0 < f' < \frac{f_s}{2} \\ &\qquad\qquad i = 1, 2, \cdots, N - 1. \end{aligned} \tag{16}$$

Let the phase characteristic $\alpha_i(f)$, $i = 1, 2, \cdots, N$, *be shaped such that*

$$\alpha_i(f) - \alpha_{i+1}(f) = \pm\frac{\pi}{2} + \gamma_i(f), \qquad f_i < f < f_i + f_s \qquad i = 1, 2, \cdots, N - 1, \tag{17}$$

where $\gamma_i(f)$ *is an arbitrary phase function with odd symmetry about* $f_i + (f_s/2)$.

If $A_i(f)$ *and* $\alpha_i(f)$ *are shaped as in (13) through (17) and* f_1 *is set according to (10), then (6), (8), (9), and (12) are simultaneously satisfied (no intersymbol or interchannel interference for a data rate of* $2f_s$ *bauds per channel). Furthermore, the five requirements in Section I are also satisfied.*

The proof of this theorem will be broken down into two parts. The first part [showing that (6), (8), (9), and (12) are simultaneously satisfied] will be given in Appendix A. The second part (showing that the five requirements in Section I are satisfied) will be given in Section III following a discussion of the various choices of the shaping functions and transmitting filter characteristics.

III. TRANSMITTING FILTER CHARACTERISTICS

Consider first the shaping of the amplitude characteristics $A_i(f)$ of the transmitting filters. Equations (13), (14), and (15) in the theorem can be easily satisfied. Equation (16) can be satisfied in many ways. For instance, a simple, practical way to satisfy (16) is stated in the following corollary.

Corollary 1: Under the simplifying condition that

(*i*) C_i *should be the same for all* i

(*ii*) $Q_i(f)$, $i = 1, 2, \cdots, N$, *should be identically shaped, i.e.,*

$$Q_{i+1}(f) = Q_i(f - f_s), \qquad i = 1, 2, \cdots, N - 1, \tag{18}$$

(16) *holds when* $Q_i(f)$ *is an even function about* f_i, *i.e.,*

$$Q_i(f_i + f') = Q_i(f_i - f'), \qquad 0 < f' < f_s . \tag{19}$$

The proof of this corollary is straightforward and need not be given here. Two examples are given for illustration purpose. The first example is illustrated in Fig. 2 where $Q_i(f)$ is chosen to be

$$Q_i(f) = \frac{1}{2}\cos\pi\frac{f - f_i}{f_s}, \qquad f_i - f_s < f < f_i + f_s \qquad i = 1, 2, \cdots, N.$$

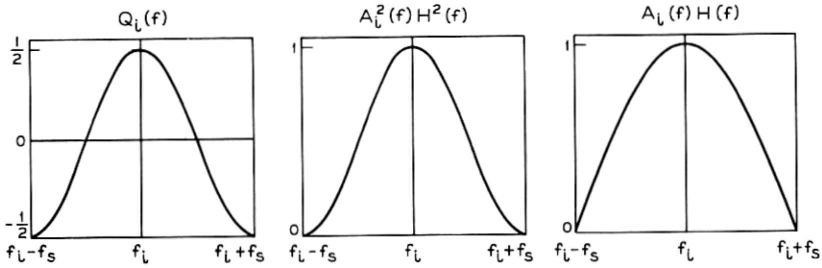

Fig. 2 — First example of shaping the amplitude characteristic $A_i(f)$ of the transmitting filters.

This simple choice satisfies (14), (15), (18), and (19). Let C_i be $\frac{1}{2}$ for all i, then (14), (15), and (16) are all satisfied. From (13)

$$A_i^2(f)H^2(f) = C_i + Q_i(f)$$
$$= \frac{1}{2} + \frac{1}{2} \cos \pi \frac{f - f_i}{f_s},$$

and

$$A_i(f)H(f) = \cos \pi \frac{f - f_i}{2f_s}, \qquad f_i - f_s < f < f_i + f_s$$
$$i = 1, 2, \cdots, N.$$

This $A_i(f)H(f)$ is similar to the amplitude characteristic of a standard duobinary filter (except shift in center frequency). The second example is illustrated in Fig. 3 where $Q_i(f)$ is chosen such that $A_i(f)H(f)$ has a shape similar to that of a multiple tuned circuit. It can be seen from these two examples that there is a great deal of freedom in choosing

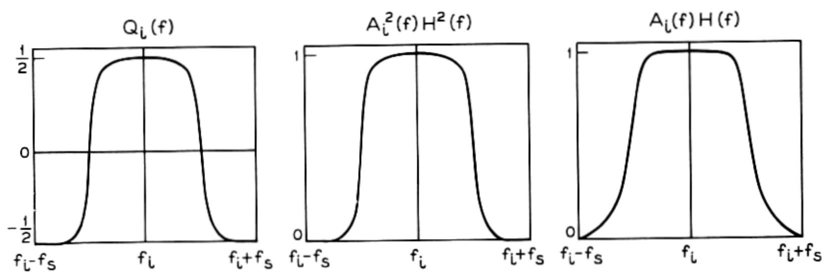

Fig. 3 — Second example of shaping the amplitude characteristic $A_i(f)$ of the transmitting filters.

the shaping function $Q_i(f)$. Consequently, $A_i(f)H(f)$ can be easily shaped into various standard forms. $A_i(f)$ would have the same shape as $A_i(f)H(f)$, if $H(f)$ is flat in the frequency band $f_i - f_s$ to $f_i + f_s$ of the ith channel. If $H(f)$ is not flat in this individual band, $A_i(f)$ can be obtained from $A_i(f)H(f)/H(f)$, provided that $H(f) \neq 0$ for any f in the band.

It is also noted from the preceding that if C_i is chosen to be the same for all i and if $Q_i(f)$, $i = 1, 2, \cdots, N$, are chosen to be identically shaped (i.e., identical in shape except shifts in center frequencies), then $A_i(f)H(f)$, $i = 1, 2, \cdots, N$, will also be identically shaped. Consequently, $A_i(f)$, $i = 1, 2, \cdots, N$, will be identically shaped if $H(f)$ is flat or is made flat. An advantage of having identically shaped filter characteristics is that each filter can be realized by using an identical shaping filter plus frequency translation.

$H(f)$ can be made flat by using a single compensating network which compensates the variation of $H(f)$ over the entire band. As an alternative, note that $A_i(f)$ exists only from $f_i - f_s$ to $f_i + f_s$. Hence, for the ith receiver, the integration limits of (6), (8), and (9) can be changed to $f_i - f_s$ and $f_i + f_s$. Therefore, the signal at the ith receiver only has to satisfy the theorem in the limited frequency band $f_i - f_s$ to $f_i + f_s$. This permits one to design the transmitting filters for flat $H(f)$ and then compensate the variation of $H(f)$ individually at the receivers, i.e., use an individual network at the ith receiver to compensate only for the variation of $H(f)$ in the limited frequency band $f_i - f_s$ to $f_i + f_s$.

Finally, note that if the channels are narrow, each channel will usually be approximately flat. In these cases, one may design the transmitting filters for flat $H(f)$ without using compensating networks. This design should lead to only small distortion.

Consider next the shaping of the phase characteristics $\alpha_i(f)$ of the transmitting filters. It is only required in the theorem that (17) be satisfied. However, if it is desired to have identically shaped transmitting filter characteristics, one may consider a simple method such as that in the following corollary.

Corollary 2: Under the simplifying condition that $\alpha_i(f)$, $i = 1, 2, \cdots, N$, *be identically shaped, i.e.,*

$$\alpha_{i+1}(f) = \alpha_i(f - f_s), \qquad i = 1, 2, \cdots, N - 1 \tag{20}$$

(17) *holds when*

$$\alpha_i(f) = h\pi \frac{f - f_i}{2f_s} + \varphi_0 + \sum_m \varphi_m \cos m\pi \frac{f - f_i}{f_s} + \sum_n \psi_n \sin n\pi \frac{f - f_i}{f_s} \tag{21}$$

$$m = 1, 2, 3, 4, 5, \cdots$$

$$n = 2, 4, 6, \cdots$$

$$f_i - f_s < f < f_i + f_s ,$$

where h is an arbitrary odd integer and the other coefficients (φ_0 ; φ_m , $m = 1, 2, 3, 4, 5, \cdots$; ψ_n , $n = 2, 4, 6, \cdots$) *can all be chosen arbitrarily.*

This corollary is proven in Appendix B. Note that if the index n in the corollary were not required to be even, $\alpha_i(f)$ would be completely arbitrary (a Fourier series with arbitrary coefficients). This shows that there is a great deal of freedom in shaping $\alpha_i(f)$ even if the additional constraint of identical shaping is introduced (three-fourths of the Fourier coefficients can be chosen arbitrarily). The linear term $h\pi[(f - f_i)/2f_s]$ is introduced not only to give the term $\pm\pi/2$ in (17), but also because a linear component is usually present in filter phase characteristics. A simple example is given in Fig. 4 to illustrate (21). For clarity, the arbitrary Fourier coefficients are all set to zero, except ψ_2 , and h is set to -1.

As can be seen in the theorem, the requirement on $\alpha_i(f)$ is independent of the requirements on $A_i(f)$. Hence, the amplitude and the phase characteristics of the transmitting filters can be synthesized independently. This gives even more freedom in designing the transmitting filters.

A simple set of $A_i(f)$ and $\alpha_i(f)$ is sketched in Fig. 5 for three adjacent channels. This illustrates that the frequency spectrum of each channel is limited and overlaps only with that of the adjacent channel. $H(f)$ is assumed flat and the transmitting filters are identically shaped. As mentioned previously, these filters can be realized either by different networks or simply by using identical shaping filters plus frequency translations.

Now consider the five requirements in Section I. The first requirement is satisfied since the transmitting filters designed are of standard forms (see the examples in Figs. 2 and 3). Perpendicular cutoffs and linear phase characteristics are not required.

As for the second requirement, it can be seen from Fig. 5(a) that the overall baseband bandwidth of N channels is $(N + 1)f_s$. Since data rate

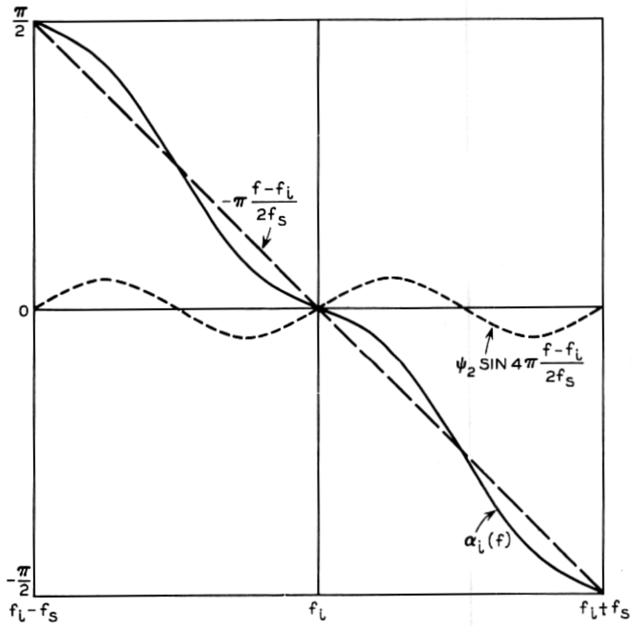

Fig. 4 — An example illustrating (21) ($h = -1$, $\psi 2 \neq 0$, all other coefficients set to zero for clarity).

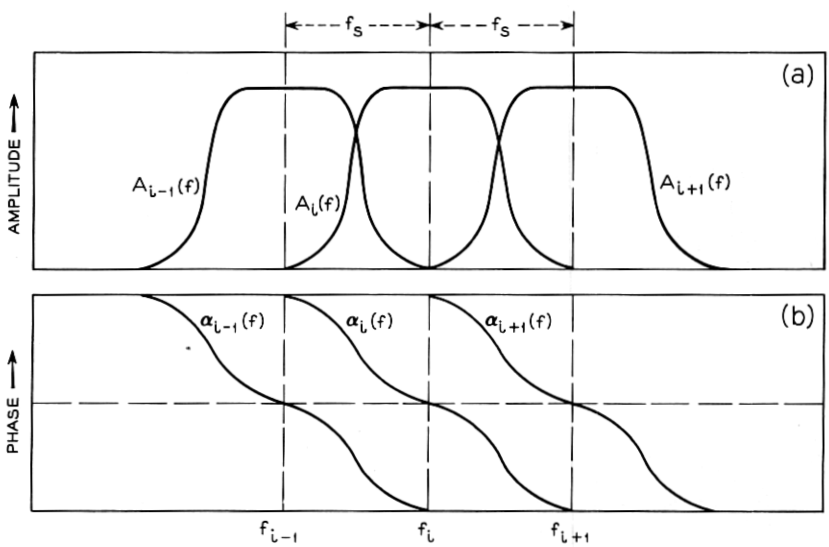

Fig. 5 — Example of transmitting filter characteristics for orthogonal multiplexing data transmission.

per channel is $2f_s$ bauds, the overall data rate of N channels is $2f_sN$ bauds. Hence,

$$\text{overall data rate} = \frac{2N}{N+1} \times \text{overall baseband bandwidth}$$
$$= \frac{N}{N+1} R_{\max},$$

where $R_{\max}$, which equals two times overall baseband bandwidth, is the Nyquist rate for which unrealizable filters with perpendicular cutoffs and linear phases are required. Thus, for moderate values of N, the overall data rate of the orthogonal multiplexing data transmission system is close to the Nyquist rate, yet rectangular filtering is not required. This satisfies requirement (*ii*).

Now consider the third requirement. As has been shown, the received pulses are orthogonal if (6), (8), and (9) are simultaneously satisfied. Note that the phase characteristic $\eta(f)$ of the transmission medium does not enter into these equations. Hence, the received signals will remain orthogonal for all $\eta(f)$, and adaptive correlation reception (see Appendix C) can be used no matter what the phase distortion is in the transmission medium. Also note that so far as each receiver is concerned, the phase characteristics of the networks in each receiver (including the bandpass filter at the input of each receiver) can be considered as part of $\eta(f)$, and hence has no effect on the orthogonality of the received signals.

In the case of the fourth requirement, let

$$b_k^i, \quad k = 0, 1, 2, \cdots; \qquad i = 1, 2, \cdots, N,$$

and

$$c_k^i, \quad k = 0, 1, 2, \cdots; \qquad i = 1, 2, \cdots, N$$

be two arbitrary distinct sets of m-ary signal digits or real numbers to be transmitted by the N AM channels. The distance in signal space between the two sets of received signals

$$\sum_i \sum_k b_k^i \cdot u_i(t - kT)$$

and

$$\sum_i \sum_k c_k^i \cdot u_i(t - kT)$$

is

$$d = \left[\int_{-\infty}^{\infty} \left[\sum_i \sum_k b_k^i \cdot u_i(t - kT) - \sum_i \sum_k c_k^i \cdot u_i(t - kT) \right]^2 dt \right]^{\frac{1}{2}}.$$

In an ideal case where interchannel and intersymbol interferences are eliminated by transmitting the signals of each channel through an independent medium and slowing down data rate such that the received signals in each channel do not overlap, the distance d can be written as

$$d_{\text{ideal}} = \left[\sum_i \sum_k \int_{-\infty}^{\infty} (b_k^{\,i} - c_k^{\,i})^2 u_i^2(t - kT)\,dt \right]^{\frac{1}{2}}.$$

In this study, the N channels transmit over the same transmission medium at the maximum data rate $T = 1/2f_s$. If the transmitting filters were not properly designed, the distance d could be much less than d_{ideal} and the system would be much more vulnerable to channel noises (impulse and Gaussian noises). However, if the transmitting filters are designed in accordance with the theorem in Section II, the received signals will be orthogonal and $d = d_{\text{ideal}}$. Thus, the distance between any two sets of received signals is preserved and the same distance protection is provided against channel noises. For instance, since $d = d_{\text{ideal}}$, it follows from maximum likelihood detection principle that for band-limited white Gaussian noise and m-ary transmission the receiver will receive each of the overlapping signals with the same probability of error as if only that signal is transmitted.

Note further that d_{ideal} can be written as

$$d_{\text{ideal}} = \left[\sum_i \sum_k (b_k^{\,i} - c_k^{\,i})^2 \int_{-\infty}^{\infty} A_i^2(f) H^2(f)\,df \right]^{\frac{1}{2}}.$$

Thus, d_{ideal} is independent of the phase characteristics $\alpha_i(f)$ of the transmitting filters and the phase characteristic $\eta(f)$ of the transmission medium. Since $d = d_{\text{ideal}}$, it follows that d is also independent of $\alpha_i(f)$ and $\eta(f)$ and the same distance protection is provided against channel noises for all $\alpha_i(f)$ and $\eta(f)$.

Finally, consider the fifth requirement. It is assumed in this paper that signaling intervals of different channels are synchronized. However, it is interesting to point out that the frequency spectra of alternate channels (for instance, $i = 1,3,5, \cdots$) do not overlap (see Fig. 5). Hence, if one uses only the odd- or the even-numbered channels, one can transmit without interchannel and intersymbol interferences and without synchronization among signaling intervals of different channels.* The overall data rate becomes $\frac{1}{2} R_{\max}$ for all N. A very attractive feature is obtained in that the transmitting filters may now have arbitrary phase

* For instance, signal digits are applied to the ith transmitting filter at 0, T, $2T$, $3T$, $\cdots$, while signal digits are applied to the $(i + 2)$th transmitting filter at τ, $T + \tau$, $2T + \tau$, $3T + \tau$, $\cdots$, where τ is an unknown constant.

characteristics $\alpha_i(f)$. [This is because $\alpha_i(f)$ is not involved in (6) and intersymbol interference is eliminated for all $\alpha_i(f)$.] Thus, only the amplitude characteristics of the transmitting filters need to be designed as in the theorem and the transmitting filters can be implemented very easily.

Another case of interest is where part of the channels are synchronized. As a simple example, assume that there are five channels and that channel 1 is synchronized with channel 2; channel 4 is synchronized with channel 5; while channel 3 cannot be synchronized with other channels. If the amplitude characteristics of the five channels plus the phase characteristics of channels 1, 2, 4, and 5 are designed as in the theorem, one can transmit simultaneously through channels, 1, 2, 4, and 5 or simultaneously through channels 1, 3, and 5 without interchannel or intersymbol interferences. The overall data rate is then between $\frac{1}{2}R_{\max}$ and $(N/N + 1)R_{\max}$.

IV. CONCLUSION

This paper presents a principle of orthogonal multiplexing for transmitting $N(N \geqq 2)$ AM data channels simultaneously through a linear band-limited transmission medium. The channels operate on equally spaced center frequencies and transmit at the same data rate with signaling intervals synchronized. Each channel can transmit binary digits, m-ary digits, or real numbers. By limiting and stacking the frequency spectrums of the channels in a proper manner, an overall data rate of

$$\frac{2N}{N+1} \times \text{overall baseband bandwidth} \qquad \text{bauds}$$

is obtained which approaches the Nyquist rate when N is large. Interchannel and intersymbol interferences are eliminated by a new method of synthesizing the transmitting filter characteristics (i.e., designing band-limited orthogonal signals). The method permits one to synthesize a large class of transmitting filter characteristics in a very convenient manner. The amplitude and the phase characteristics can be synthesized independently. The transmitting filter characteristics obtained are practical in that

(*i*) The amplitude characteristics may have gradual rolloffs, and the phase characteristics need not be linear.

(*ii*) The transmitting filters may be identically shaped and can be realized simply by identical shaping filters plus frequency translations.

It is noted that the principle presented in this paper uses band-limited orthogonal signals as opposed to other orthogonal multiplexing schemes using nonband-limited orthogonal signals. The chief advantage of using band-limited signals is that (as mentioned in Section I) these signals can be transmitted through a band-limited transmission medium at a maximum data rate without interchannel and intersymbol interferences. Other advantages of using band-limited signals over methods using nonband-limited signals are

(*i*) Permitting the use of a narrowband bandpass filter at the input of each receiver (see Appendix C) to reject noises and signals outside the band of interest. This is particularly important in suppressing impulse noises and in preventing overloading the front ends of the receivers.

(*ii*) Permitting unsynchronized operations at data rates between $\frac{1}{2}R_{\max}$ and $(N/N+1)R_{\max}$.

It has been shown that the received signals remain orthogonal for all phase characteristics of the transmission medium; hence, adaptive correlation reception can be used to separate the received signals no matter what the phase distortion is in the transmission medium. These correlators adapt not only to the phase distortions in the system (including transmission medium, bandpass receiving filters, etc.), but also (see Appendix C) to the phase difference between modulation and demodulation carriers (easing synchronization requirements).

V. ACKNOWLEDGMENTS

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APPENDIX A

In this appendix, it will be proven that if the transmitting filters $A_i(f) \exp [J\alpha_i(f)]$, $i = 1, 2, \cdots, N$, are shaped as in the theorem in Section II and f_1 is set according to (10), then equations (6), (8), (9), and (12) are simultaneously satisfied.

First consider (6). From (13)

$$\int_0^{\infty} A_i^2(f)H^2(f) \cos 2\pi fkT \, df = \int_{f_i - f_s}^{f_i + f_s} [C_i + Q_i(f)] \cos 2\pi fkT \, df. \quad (22)$$

Since $T = 1/2f_s$, one has

$$\int_{f_i-f_s}^{f_i+f_s} C_i \cos 2\pi fkT \, df = \frac{C_i}{2\pi kT} [\sin 2\pi(f_i + f_s)kT - \sin 2\pi(f_i - f_s)kT]$$
$$= \frac{C_i}{\pi kT} \sin \pi k \cos 2\pi f_i kT \tag{23}$$
$$= 0, \qquad k = 1, 2, 3, \cdots \qquad i = 1, 2, 3, \cdots, N.$$

Since $f_i = (h + i - \frac{1}{2})f_s$, one has

$$2\pi \left(f_i - \frac{f_s}{2}\right) kT = 2\pi(h + i - 1)f_s k \frac{1}{2f_s} = (h + i - 1)k\pi. \tag{24}$$

Hence, $\cos 2\pi fkT$ is an even function about $f_i - (f_s/2)$. This, together with the fact that $Q_i(f)$ is an odd function about $f_i - (f_s/2)$ [see (15)], gives

$$\int_{f_i-f_s}^{f_i} Q_i(f) \cos 2\pi fkT \, df = 0, \qquad k = 1, 2, 3, \cdots \qquad i = 1, 2, \cdots, N. \tag{25}$$

Similarly, one can show

$$\int_{f_i}^{f_i+f_s} Q_i(f) \cos 2\pi fkT \, df = 0, \qquad k = 1, 2, 3, \cdots \qquad i = 1, 2, \cdots, N. \tag{26}$$

Substituting (23), (25), and (26) into (22) gives

$$\int_0^\infty A_i^2(f)H^2(f) \cos 2\pi fkT \, df = 0, \qquad k = 1, 2, 3, \cdots \qquad i = 1, 2, 3, \cdots, N.$$

Thus, (6) is satisfied and intersymbol interference is eliminated.

Next consider interchannel interference and (8) and (9). From (13),

$$A_i(f)H(f) = 0, \qquad f < f_i - f_s, f > f_i + f_s,$$

so

$$A_i(f)A_j(f)H^2(f) = 0 \quad \text{for} \quad j = i \pm 2, i \pm 3, i \pm 4, \cdots,$$

or

$$\int_0^\infty A_i(f)A_j(f)H^2(f) \cos [\alpha_i(f) - \alpha_j(f)] \cos 2\pi fkT \, df = 0$$

$$\int_0^\infty A_i(f)A_j(f)H^2(f) \sin [\alpha_i(f) - \alpha_j(f)] \sin 2\pi fkT \, df = 0 \tag{27}$$

$$k = 0, 1, 2, \cdots$$
$$i = 1, 2, 3, \cdots, N$$
$$j = i \pm 2, i \pm 3, i \pm 4, \cdots .$$

Equation (27) shows that (8) and (9) are satisfied for $j = i \pm 2, i \pm 3, i \pm 4, \cdots$. It remains to show that (8) and (9) hold for $j = i \pm 1$. Consider $j = i + 1$. It is seen from (13) that

$$\begin{aligned} A_i(f)A_{i+1}(f)H^2(f) &= [C_i + Q_i(f)]^{\frac{1}{2}}[C_{i+1} + Q_{i+1}(f)]^{\frac{1}{2}}, \qquad f_i < f < f_i + f \\ &= 0, \qquad f < f_i, f > f_i + f_s . \end{aligned} \tag{28}$$

One can write from (17) and (28)

$$\begin{aligned} &\int_0^\infty A_i(f)A_{i+1}(f)H^2(f) \cos [\alpha_i(f) - \alpha_{i+1}(f)] \cos 2\pi fkT \, df \\ &= \int_{f_i}^{f_i+f_s} [C_i + Q_i(f)]^{\frac{1}{2}}[C_{i+1} + Q_{i+1}(f)]^{\frac{1}{2}} \cos \left[\pm\frac{\pi}{2} + \gamma_i(f)\right] \\ &\qquad \cdot \cos 2\pi fkT \, df \end{aligned} \tag{29}$$

$$k = 0, 1, 2, \cdots$$
$$i = 1, 2, \cdots, N.$$

It is required in the theorem that

$$[C_i + Q_i(f)]^{\frac{1}{2}}[C_{i+1} + Q_{i+1}(f)]^{\frac{1}{2}}$$

be an even function about $f_i + (f_s/2)$. Furthermore, $\cos [\pm(\pi/2) + \gamma_i(f)]$ and $\cos 2\pi fkT$ are, respectively, odd and even functions about $f_i + (f_s/2)$. Hence, from (29)

$$\int_0^\infty A_i(f)A_{i+1}(f)H^2(f) \cos [\alpha_i(f) - \alpha_{i+1}(f)] \cos 2\pi fkT \, df = 0$$

$$k = 0, 1, 2, \cdots \tag{30}$$
$$i = 1, 2, \cdots, N.$$

Equation (30) shows that (8) is satisfied for $j = i + 1$. In a similar manner, one can show that (8) holds for $j = i - 1$ and that (9) holds for $j = i \pm 1$. These, together with (27), prove that (8) and (9) hold for all k, i, and j.

APPENDIX B

Proof of Corollary 2

From (20) and (21)

$$\begin{aligned} \alpha_{i+1}(f) &= \alpha_i(f - f_s) \\ &= h\pi \frac{f - f_s - f_i}{2f_s} + \varphi_0 \\ &\quad + \sum_m \varphi_m \cos m\pi \frac{f - f_s - f_i}{f_s} \\ &\quad + \sum_n \psi_n \sin n\pi \frac{f - f_s - f_i}{f_s} \\ m &= 1, 2, 3, 4, 5, \cdots \\ n &= 2, 4, 6, \cdots \\ f_i &< f < f_i + 2f_s . \end{aligned} \tag{31}$$

For $f_i < f < f_i + f_s$, one has from (21) and (31)

$$\begin{aligned} \alpha_i(f) - \alpha_{i+1}(f) = \frac{h\pi}{2f_s} [f - f_i - (f - f_s - f_i)] \\ + \sum_m \varphi_m \left[\cos m\pi \frac{f - f_i}{f_s} \right. \\ \left. - \cos m\pi \frac{f - f_s - f_i}{f_s} \right] \\ + \sum_n \psi_n \left[\sin n\pi \frac{f - f_i}{f_s} \right. \\ \left. - \sin n\pi \frac{f - f_s - f_i}{f_s} \right] \end{aligned}$$

$$\begin{aligned} &= \frac{h\pi}{2} + \sum_m \varphi_m \left[-2 \sin \frac{1}{2} \left(m\pi \frac{2f - 2f_i - f_s}{f_s} \right) \sin \frac{m\pi}{2} \right] \\ &\qquad + \sum_n \psi_n \left[2 \sin \frac{n\pi}{2} \cos \frac{1}{2} \left(n\pi \frac{2f - 2f_i - f_s}{f_s} \right) \right] \\ &= \frac{h\pi}{2} - 2 \sum_l \varphi_l \sin \frac{l\pi}{2} \sin \frac{l\pi(2f - 2f_i - f_s)}{2f_s} \\ &\qquad\qquad l = 1, 3, 5, \cdots \\ &\qquad\qquad h = \pm 1, \pm 3, \cdots . \end{aligned} \tag{32}$$

Since $\sin [l\pi(2f - 2f_i - f_s)/2f_s]$ is an odd function about $f = f_i + (f_s/2)$, (32) is equivalent to (17) and corollary 2 is proven.

APPENDIX C

This appendix briefly describes a possible receiver structure for receiving the multichannel orthogonal signals.

The receiver of a single channel (say, the fifth channel) is shown in Fig. 6(*a*). When viewed at point B toward the transmitter, the channels have amplitude characteristics as shown in Fig. 6(b). The bandpass filter at the input of the fifth receiver has a passband from $f_5 - f_s$ to $f_5 + f_s$ [Fig. 6(c)]. This filter serves the important purpose of rejecting noises and signals outside the band of interest. Sharp impulse noises with broad frequency spectra are greatly attenuated by this filter. Signals in other channels are rejected to prevent overloading and cross modulation.

The product device translates the frequency spectra further toward the origin so that the signal can be represented by a minimum number of accurate time samples and the adaptive correlator can operate in digital fashion. The transmitter can transmit a reference frequency f_s or a known multiple of f_s to the receivers for deriving the signals $\cos [2\pi(i - 1)f_s t + \theta_i]$ for the product devices. It is important to note that the transmitter can lock this frequency f_s to the data rate $2f_s$ so that the arbitrary phase angle θ_i is time invariant and can be taken into account by adaptive correlation. Furthermore, the receiver can also derive the sampling rate $2f_s$ from this reference frequency.

When observed at point D, the channels have amplitude characteristics as shown in Fig. 6(d). Note that the fifth channel now has a center frequency at $1.5f_s$ [satisfying (11)] and an undistorted amplitude characteristic; hence, the signals in channel 5 remain orthogonal. The overlapping frequency spectra between channel 5 and channels 4 and 6 remain undistorted, and the phase differences $\alpha_4(f) - \alpha_5(f)$ and $\alpha_5(f) - \alpha_6(f)$ are unchanged; therefore, the signals in channels 4 and 6 remain

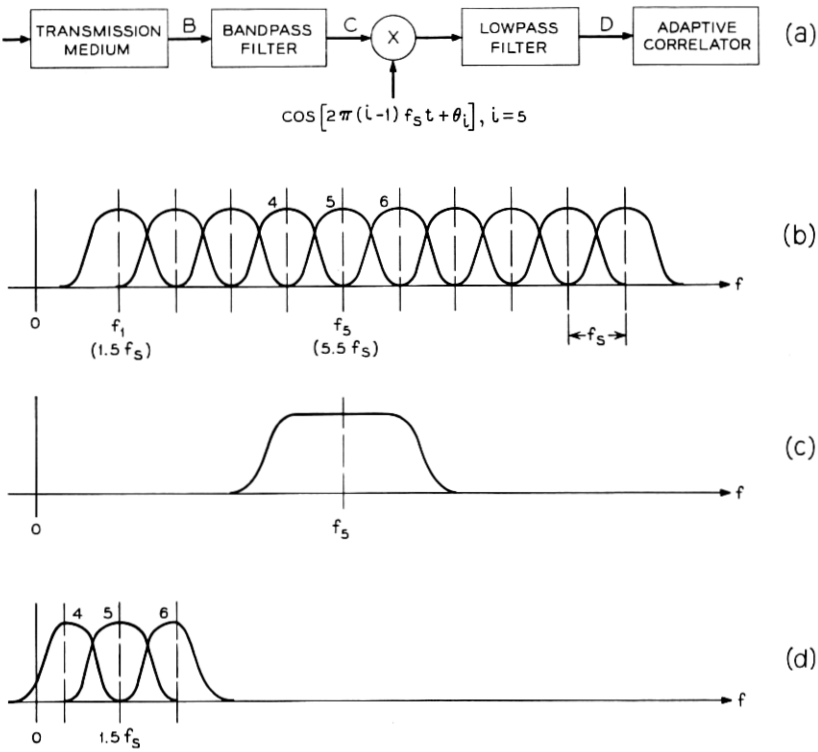

Fig. 6 — Reception of the signals in channel 5.

orthogonal to those in channel 5. Other channels produce no interference since their spectra do not overlap with that of channel 5.

Let $b_0u(t)$, $b_1u(t - T)$, $b_2u(t - 2T)$, $\cdots$ be the signals in channel 5 at point D, where b_0, b_1, b_2, $\cdots$ are the information digits. These signals can be represented by vectors of time samples as

$$b_0\underline{u}_0 , \; b_1\underline{u}_1 , \; b_2\underline{u}_2 , \; \cdots .$$

Since $u(t)$, $u(t - T)$, $\cdots$ differ only in time origin, it is only necessary to learn $\underline{u}_0$ for correlation purposes. The received signal at point D can be written as

$$\sum_n b_n\underline{u}_n + \underline{v},$$

where $\underline{v}$ represents the sum of the signals in other channels. From discussions in the preceding paragraph

$$\begin{aligned} \underline{u}_k'\underline{u}_j &= \lambda \qquad k = j \\ &= 0 \qquad k \neq j \\ u_k'\underline{v} &= 0. \end{aligned}$$

Thus, the adaptive correlator can learn the vector $\underline{u}_0$ prior to data transmission and then correlate the received signal with $\underline{u}_k$, $k = 0, 1, 2, \cdots$ to obtain the information digits b_k, $k = 0, 1, 2, \cdots$.

In order to describe the operation more clearly we assume that the signal at point D is fed to a delay line tapped at $T/3$-second intervals (signal at D is band-limited between 0 and $3f_s$). Assuming that $u(t)$ is essentially time-limited to mT seconds for all possible phase characteristics of the transmission system, then $3m$ taps are sufficient. The ith tap is connected to a gain control G_i. In the training period prior to data transmission, the ith tap is also connected to a sampler s_i. In the training period, the transmitter transmits a series of identical test pulses at $t = 0$, lT, $2lT$, $\cdots$. The integer l is chosen large enough such that the received test pulses $u(t)$, $u(t - lT)$, $u(t - 2lT)$, $\cdots$ do not overlap. The sampler s_i samples at $t = \tau$, $lT + \tau$, $2lT + \tau$, $\cdots$. The only requirement on τ is that $u(t)$ should be approximately centered on the tapped delay line at $t = \tau$. The output of s_i (without noise) is a series of samples each representing the ith time sample u_i of $u(t)$. Since noise is always present, these samples are passed through a network (probably a simple RC circuit) such that the output $\hat{u}_i$ of this network is an estimate of $u_i \cdot \hat{u}_i$ is in the form of a voltage or current and hence can be used to set the gain control G_i of the ith tap. Thus, at the end of the training period, the gain controls of the successive taps are set according to the magnitudes of the successive time samples of $u(t)$.

During data transmission, the transmitter transmits the information digits b_0, b_1, b_2, $\cdots$ sequentially at $t = 0$, T, $2T$, $\cdots$. A sampler at the receiver samples the sum of the outputs of all the tap gain controls at $t = \tau$, $T + \tau$, $2T + \tau$, $\cdots$ to recover b_0, b_1, b_2, $\cdots$. The time delay τ remains the same as in the training period. The data transmission operates in real time.

REFERENCES

1. Mosier, R. R., A Data Transmission System Using Pulse Phase Modulation, IRE Convention Record of First National Convention on Military Electronics, June 17–19, 1957, Washington, D. C.
2. *Dynamic Error-Free Transmission,* General Dynamics/Electronics—Rochester, N. Y.
3. Harmuth, H. F., On the Transmission of Information by Orthogonal Time Functions, AIEE Trans., pt. I (Communication and Electronics), July, 1960, pp. 248–255.
4. Bennett, W. R. and Davey, J. R., *Data Transmission,* McGraw-Hill Book Company, Inc., New York, 1965, p. 65.